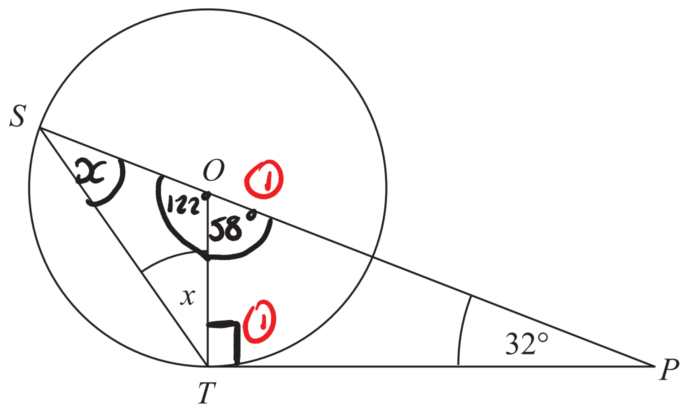
1.



S and T are points on the circumference of a circle, centre O.
PT is a tangent to the circle.
SOP is a straight line.
Angle $OPT = 32°$

Work out the size of the angle marked x.
You must give a reason for each stage of your working.

Tangents and radius meet at 90° so angle PTO is 90°

Interior angles of triangle add to 180°

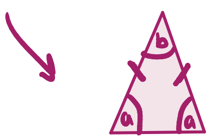
$180 - 32 - 90 = 58$ so angle POT is 58°

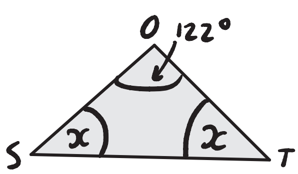
Angles at a point on a straight line add to 180°

$180 - 58 = 122$ so angle SOT is 122°

OS and OT are both the radius so are the same length meaning triangle SOT must be isosceles

∴ angle TSO is x





1 Reasoning

Interior angles of triangle add to 180°

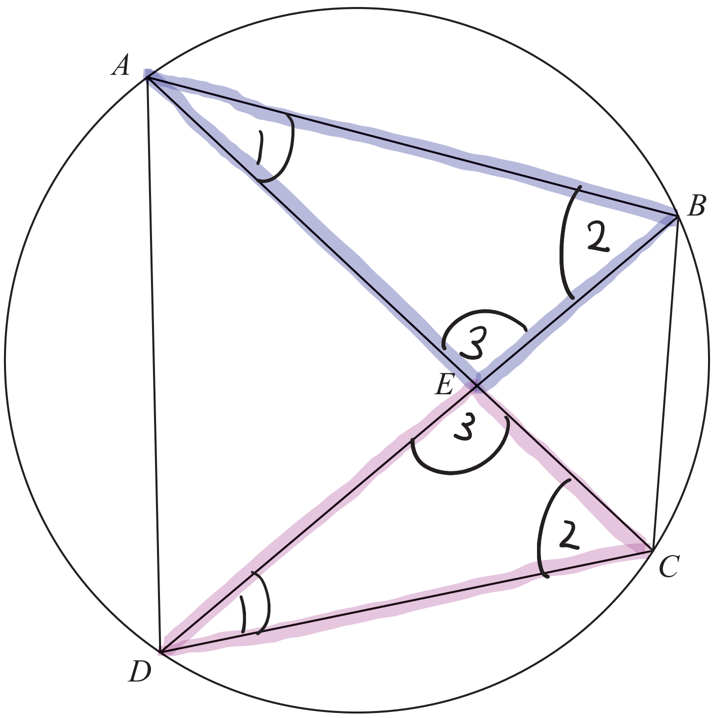
$x + x + 122 = 180$

$2x + 122 = 180$

$2x = 58$

$x = 29°$ 1

2. *A*, *B*, *C* and *D* are four points on the circumference of a circle.



AEC and *BED* are straight lines.

Prove that triangle *ABE* and triangle *DCE* are similar.
You must give reasons for each stage of your working.

→ Two triangles are similar if their side lengths are all in the same ratio.

∠BAC = ∠EDC because angles in the same segment are equal (1)

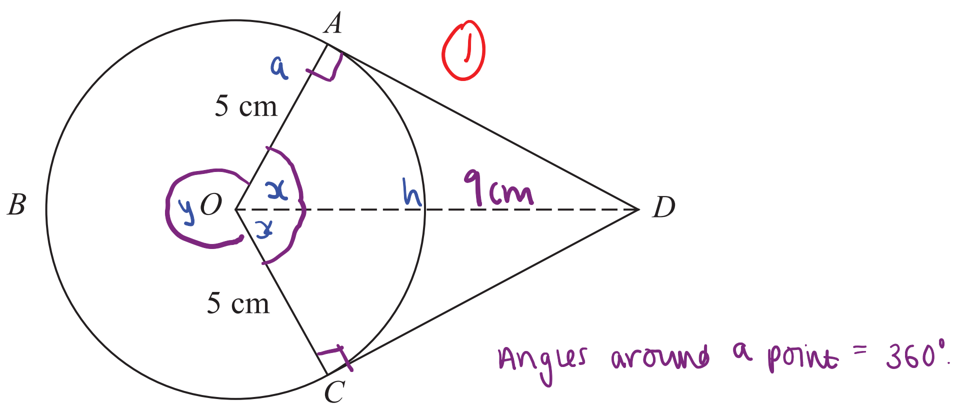
∠ABE = ∠ECD because angles in the same segment are equal.

∠AEB = ∠DEC because vertically opposite angles are equal. (1)

∴ Triangles ABE and DCE are similar because they have three pairs of equal angles. (1)
(BAC = EDC, ABE = ECD, AEB = DEC.)

(Total for Question is 3 marks)

3.



A, *B* and *C* are points on a circle of radius 5 cm, centre *O*.
DA and *DC* are tangents to the circle.
DO = 9 cm

Work out the length of arc *ABC*.
Give your answer correct to 3 significant figures.

Triangle AOD: $\cos x = \frac{a}{h} = \frac{5}{9}$ (1)

$x = \cos^{-1}\left(\frac{5}{9}\right) = 56.2510114°$ (1)

$y = 360 - (x + x) = 360 - 2x$

$= 360 - (2 \times 56.2510114)$

$y = 247.4979772°$

Arc ABC: $\frac{\theta}{360} \times (2\pi r)$

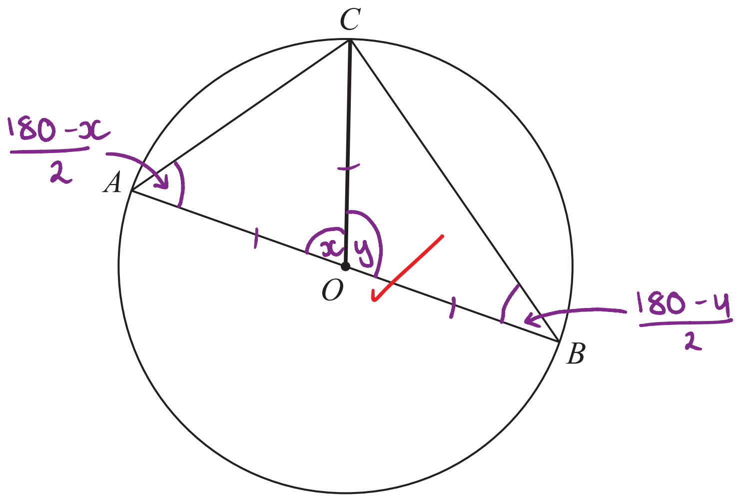
$= \left(\frac{247.49\ldots}{360}\right) \times (2 \times \pi \times 5)$ (1)

$= 21.59827\ldots \text{ cm} = \underline{\underline{21.6 \text{ cm}}}$ (1)

21.6 cm

(Total for Question is 5 marks)

4.



A, *B* and *C* are points on the circumference of a circle, centre *O*.
AOB is a diameter of the circle.

Prove that angle *ACB* is 90°
You must **not** use any circle theorems in your proof.

Angles on straight line add to 180°

$\therefore x + y = 180°$

$2 \times \angle CAO + x = 180$
$(-x) \quad (-x)$
$2 \times \angle CAO = 180 - x$
$(\div 2) \quad (\div 2)$
$\angle CAO = \frac{180 - x}{2}$

$\frac{180-x}{2} + \frac{180-y}{2} + \angle ACB = 180$

$90 - \frac{x}{2} + 90 - \frac{y}{2} + \angle ACB = 180$

$180 - \frac{x}{2} - \frac{y}{2} + \angle ACB = 180$

$180 - \frac{1}{2}(x+y) + \angle ACB = 180$

If $x + y = 180$

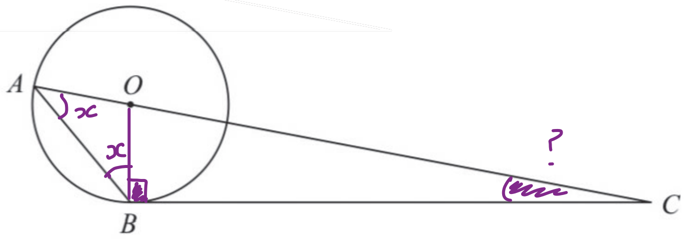
$180 - \frac{1}{2}(180) + \angle ACB = 180$

$180 - 90 + \angle ACB = 180$

$\angle ACB = 90°$

(Total for Question is 4 marks)

5.



A and B are points on a circle, centre O.

BC is a tangent to the circle.
AOC is a straight line.
Angle $ABO = x°$.

Find the size of angle ACB, in terms of x.
Give your answer in its simplest form.
Give reasons for each stage of your working.

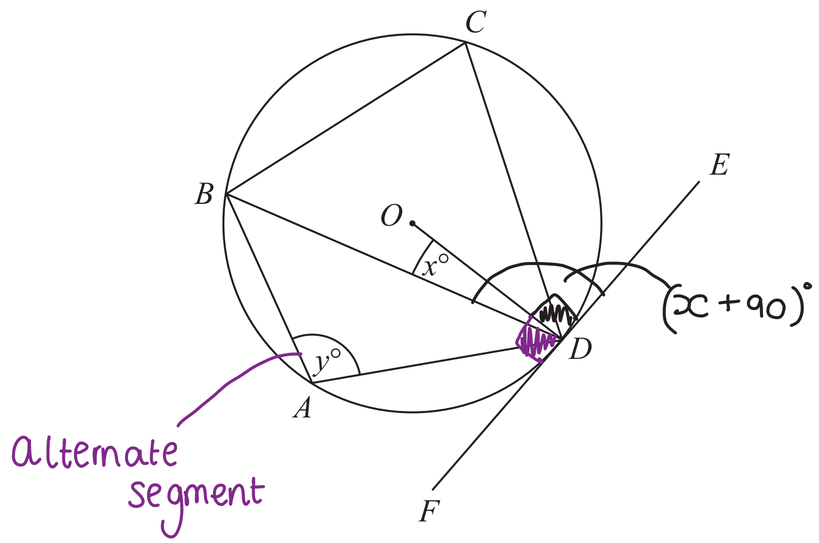
Finding $\angle ABC$:
The tangent to a circle is perpendicular to its radius (1)
$OB \perp BC$ $\therefore \angle OBC = 90°$
$\therefore \angle ABC = x + 90$ (1)

Finding $\angle OAB$:
$AO = OB =$ radius of circle
Triangle AOB has 2 sides of equal length so is isosceles
The base angles in an isosceles triangle are equal (1)
$\therefore \angle ABO = \angle OAB = x$
so $\angle OAB = x°$

Finding $\angle ACB$:
Angles in a triangle sum to 180°
$\angle ACB + \angle CAB + \angle ABC = 180°$
$\angle ACB = 180 - \angle CAB - \angle ABC$ (1)
$= 180 - x - (90 + x)$
$= 90 - 2x$ (1)

(Total for Question is 5 marks)

6.



A, B, C and D are points on the circumference of a circle, centre O.
FDE is a tangent to the circle.

(a) Show that $y - x = 90$
You must give a reason for each stage of your working.

The tangent of the circle is perpendicular to its radius

$\therefore \angle ODE = \angle ODF = 90$ 1

$\angle BDE = x + \angle ODE$
$= x + 90$

$\angle BDE = y$ due to the alternate segment theorem

1 correct circle theorems for chosen method

$\therefore \angle BDE = y = x + 90$ $y = x + 90$

$y - x = 90$ as required 1

(3)

Dylan was asked to give some possible values for x and y.

He said,
"y could be 200 and x could be 110, because 200 – 110 = 90"

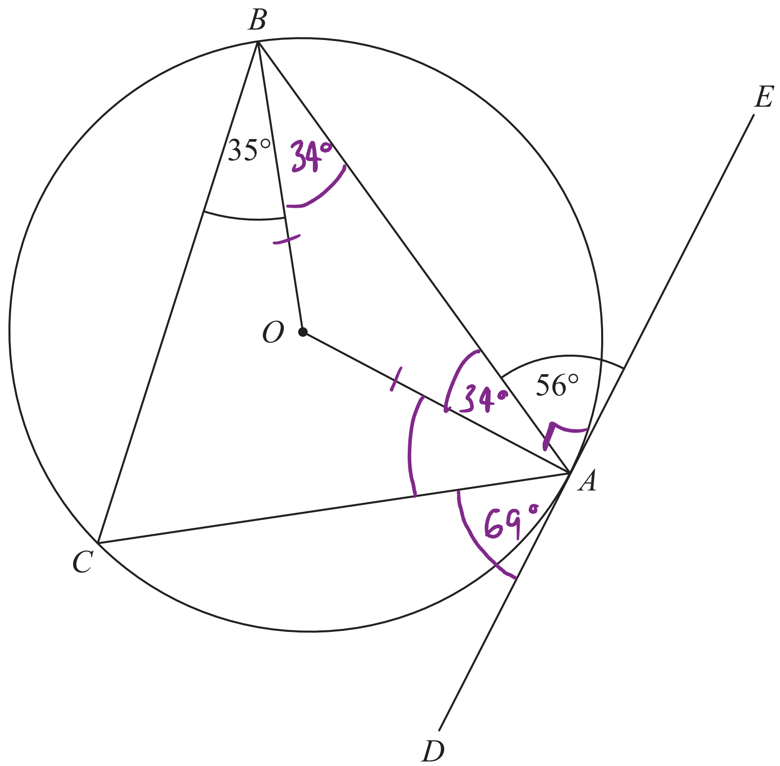
(b) Is Dylan correct?
You must give a reason for your answer.

No, y must be less than 180 as it is an angle in a triangle

(1)

(Total for Question is 4 marks)

7.



A, B and C are points on the circumference of a circle, centre O.
DAE is the tangent to the circle at A.

Angle $BAE = 56°$
Angle $CBO = 35°$

Work out the size of angle CAO.
You must show all your working.

$\angle BAO = 90 - 56$
$= 34°$

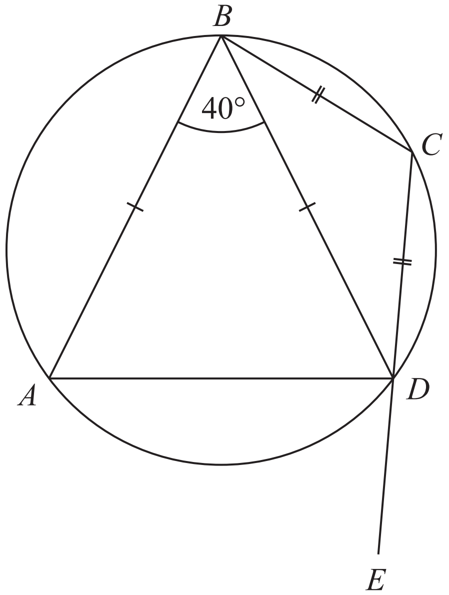
$\angle DAC = 35 + 34$
$= 69°$

$\angle CAO = 180 - 69 - 34 - 56$
$= 21°$

21° °

(Total for Question is 3 marks)

8. The points A, B, C and D lie on a circle.
CDE is a straight line.



$BA = BD$
$CB = CD$
Angle $ABD = 40°$

Work out the size of angle ADE.
You must give a reason for each stage of your working.

BA = BD ∴ ΔBAD is isosceles.
Base angles are equal.

Angles in a Δ = 180° (1) (1)

∴ $B\hat{A}D = B\hat{D}A = \frac{180-40}{2} = 70°$

Opposite angles of a cyclic quadrilateral = 180° (1)

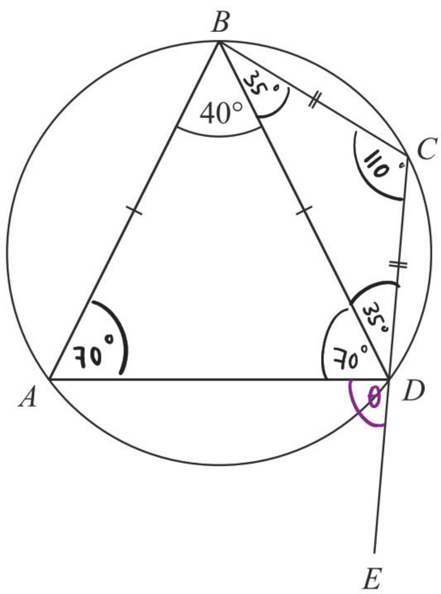
∴ $B\hat{C}D = 180 - 70 = 110°$ (1)

CB = CD ∴ ΔBCD is isosceles.

∴ $C\hat{B}D = C\hat{D}B = \frac{180-110}{2} = 35°$

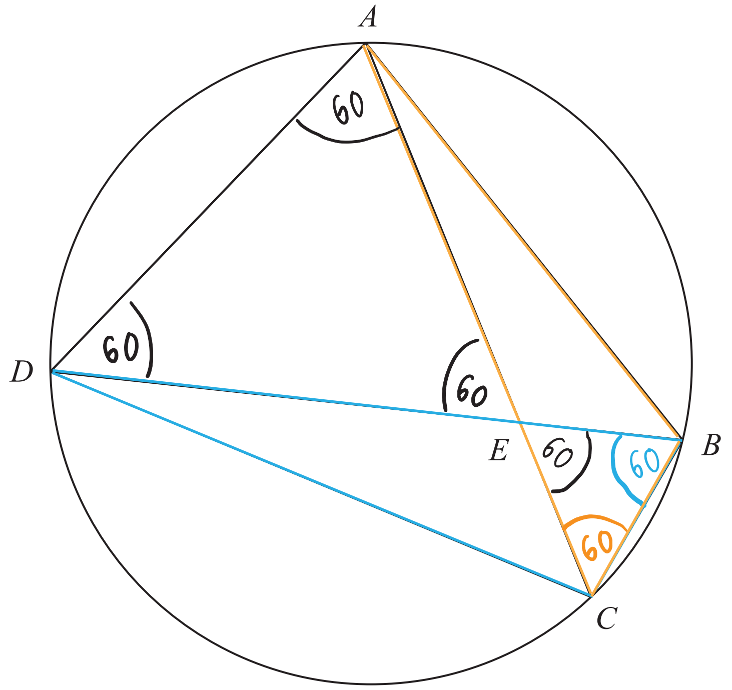
Angles in a straight line add up to 180°

∴ $A\hat{D}E = 180 - (35 + 70) = \underline{\underline{75°}}$ (1)



(Total for Question is 5 marks)

9. A, B, C and D are four points on a circle.



AEC and DEB are straight lines.

Triangle AED is an equilateral triangle. → SSS, ASA, SAS, RHS.

Prove that triangle ABC is congruent to triangle DCB.

Line BC is shared by both triangles. (1)

AED is equilateral ∴ ∠AED = ∠ADE = ∠DAE = 60° (1)

∠DAC = ∠DBC because angles in the same segment are equal.

∠ADB = ∠ACB because angles in the same segment are equal.

∴ ∠ACB = ∠DBC (1)

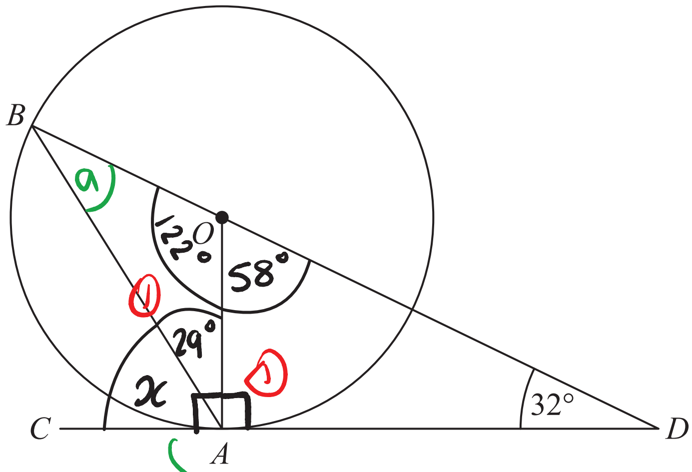
∠CEB = 60 ∴ ΔEBC is equilateral

AC = AE + EC = DE + EB = DB. ∴ AC = DB (1)

$\boxed{\text{ΔABC is congruent to ΔDCB because they meet the SAS criteria.}}$

(Total for Question is 4 marks)

10.



A and B are points on a circle with centre O.
CAD is the tangent to the circle at A.
BOD is a straight line.

Angle $ODA = 32°$

Work out the size of angle CAB.
You must show all your working.

$x + 29 = 90$
$x = 90 - 29$
$x = 61°$ (1)

Since sum of all interior angles in a triangle = 180°

Angle AOD = 180 − 90 − 32 = 58°

Since angles on a straight line add to 180°

Angle AOB = 180 − 58 = 122°

Since we know triangle AOB is isosceles since OA and OB are the same length

$2a + 122 = 180$
$2a = 180 - 122$
$a = \frac{1}{2}(180 - 122) = 29$

................61................ °

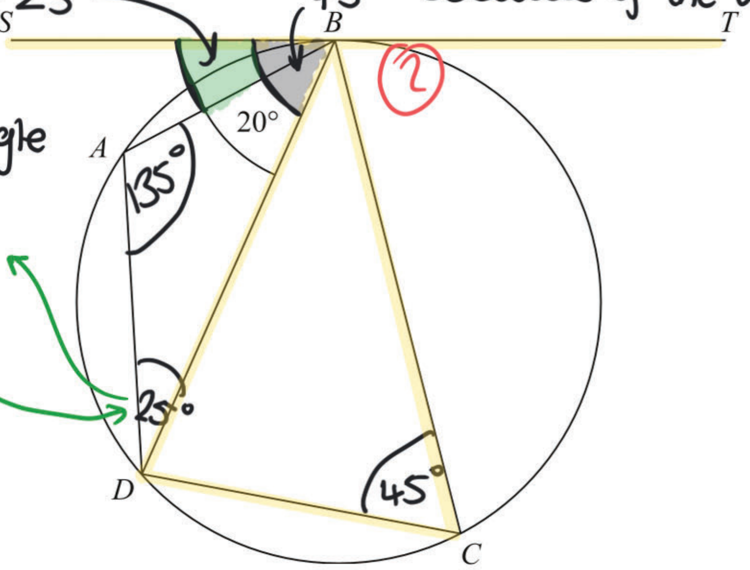
11.

$45 - 20 = 25°$

$45°$ because of the alternate segment theorem

Angles in triangle add to $180°$

$180 - 20 - 135$
$= 25°$



A, *B*, *C* and *D* are four points on a circle.
SBT is a tangent to the circle.
Angle *ABD* = 20°

the size of angle *BAD* : the size of angle *BCD* = 3 : 1

Find the size of angle *SBA*.
Give a reason for each stage of your working.

Opposite angles in a cyclic quadrilateral add to $180°$ 1

$180°$ ← Share in ratio 3:1

$\frac{180}{3+1} = \frac{180}{4} = 45°$ ← One 'part' of the ratio

angle $BAD = 3 \times 45 = 135°$
angle $BCD = 1 \times 45 = 45°$

25 1 °